737. Presentation of the concept of stability of the hybrid powertrain by the Lyapunov theory

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Abstract. The paper presents an algorithm for controlling a hybrid drive system equipped with an internal combustion engine and electric motor. Work algorithm provides basic control of the petrol engine as the primary energy source. The internal combustion engine control system adopted an adaptive control system, which taking into account the stability of the system by Lyapunov allows for optimum combustion engine and electric motor. The direct method by Lyapunov stability analysis allows one to assess the stability of the whole system of hybrid propulsion system of a wheeled vehicle.

Keywords: Lyapunov stability, hybrid powertrain system, wheeled vehicle.

Introduction

The basis for a wheeled vehicle traffic in a complex physical system is the external forces (disturbances) and the layout of the internal forces (in the form of power and torque, the forces associated with the intention of steering a wheeled vehicle, affected by the action of the driver) running on that vehicle. There have been many works including an analysis of design, technology and operation of wheeled vehicles. Today, many research centers and car companies focus their efforts on the marketing of environmentally friendly vehicles. Most effective vehicles in terms of generated power and energy are the vehicles with hybrid powertrain. These vehicles have very good driving properties and high energy density accumulated in the fuel. Gasoline supplies as fuel engine (burning). Energy produced is used to drive an electric generator and vehicle traffic.

Due to the rapid development of hybrid vehicles was a lot of work in this field, but there is no information related to the stability of work of such mechatronic system. Therefore it was decided to fill this gap and provide stability of the hybrid propulsion system having a direct method for solving systems of nonlinear in the sense of Lyapunov. This kind of comprehensive analysis and synthesis of theory depends on the actual conditions of the propulsion system. With such conditions theoretical analysis may enable to predict the behavior of the hybrid propulsion system as a mechatronic system during its operation and to identify deviations from the ideal (programmed) motion and to assess the regularity of movement and operation of such a propulsion system. Theoretical analysis includes examining the stability of mechatronic motion in more specific terms of work and provided for its operation [1, 2, 4, 6].

Mathematical models

By analyzing the stability assessment propulsion systems, it is possible to design characteristics of the dynamical system. Such a system upset back to equilibrium after the disappearance of extortion. Therefore, it is necessary to study the stability of the propulsion system (in particular, the hybrid powertrain) due to:

- external forces acting on wheeled vehicles (propulsion system) which cause distortion of the propulsion system in its motion by changing the forces of resistance to motion,
- internal forces, i.e. forces that directly depend on the actions of the driver, for example, increasing the throttle opening increases the torque and thus increase the fixed traffic disruption,
- the movement of a wheeled vehicle on which it becomes necessary to take into account the

working conditions of the individual drive units, each with separate wheeled vehicle can drive through the synergy of energy or drive the wheeled vehicles at the same time,

- aspect of the internal combustion engine driving an electric generator.

With the above assumptions, the working conditions with the following engines are investigated:

- wheeled vehicle moves only on the unit of electricity,
- wheeled vehicle moves only on electric power at low line speeds (urban agglomerations),
- internal combustion engine driving an electric generator operated at maximum efficiency (the smallest unit of fuel consumption - BSFC),
- a combustion engine assisted by an electric motor, where:
  - a required torque demand - the signal comes from the driver's throttle in the form of >0.9,
  - the required demand for power - the forces of resistance to motion are greater than the received power of an electric motor.

When linearizing the description of the mathematical model of the propulsion system around a fixed point, we get the opportunity to examine the stability of the local analytically. The problem of global stability testing of the propulsion system is the subject much more complicated, thus it requires a greater effort of research. The paper presents a test of stability by limiting the local stability analysis for AC-powered motor with frequency converter - an analysis of the object controlled [3].

The propulsion system made of alternating current motor is examined in terms of stability only after the division of the following types (Fig. 1):

- system associated with frequency,
- the control system of electric motor operation (control system),
- control system (electric motor).

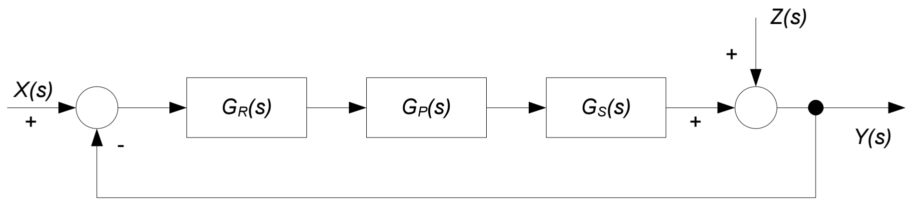

Fig. 1. Block structure of the drive-train system of AC-motor: $X(s)$ – input, $Y(s)$ – output, $Z(s)$ – distorting signal, $G_R(s)$ – control system, $G_P(s)$ – inverter, $G_S(s)$ – electric motor

Since the method of alternating current motor control depends on the control system transfer function, which is why choosing the right control system ensures stable operation of an electric propulsion system and the dynamism of change volume adjustable.

For the test, the mathematical model of three-phase motor in the form of differential equations in the *d-q* coordinate system, associated with the rotor flux spatial vector [3]:

$$\frac{di_{s(d)}}{d\tau} = \frac{R_s L_r^2 + R_r L_m^2}{L_r \omega_\delta} i_{s(d)} + \frac{R_r L_m}{L_r \omega_\delta} \psi_{r(d)} + \omega_a i_{s(q)} + \omega_r \frac{L_m}{\omega_\delta} \psi_{r(q)} + \frac{L_r}{\omega_\delta} u_{s(d)} \tag{1}$$

$$\frac{di_{s(q)}}{d\tau} = \frac{R_s L_r^2 + R_r L_m^2}{L_r \omega_\delta} i_{s(q)} + \frac{R_r L_m}{L_r \omega_\delta} \psi_{r(q)} - \omega_a i_{s(d)} - \omega_r \frac{L_m}{\omega_\delta} \psi_{r(d)} + \frac{L_r}{\omega_\delta} u_{s(q)} \tag{2}$$

$$\frac{d\psi_{r(d)}}{d\tau} = \frac{R_r}{L_r} \psi_{r(d)} + (\omega_a - \omega_r) \psi_{r(q)} + \frac{R_r L_m}{L_r} i_{s(d)} + u_{r(d)} \tag{3}$$

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$$\frac{d\psi_{r(q)}}{d\tau} = -\frac{R_r}{L_r}\psi_{r(q)} - (\omega_a - \omega_r)\psi_{r(d)} + \frac{R_r L_m}{L_r} i_{s(q)} + u_{r(q)} \tag{4}$$

$$\frac{d\omega_r}{d\tau} = \frac{L_m}{IL_r}(\psi_{r(d)} i_{s(q)} - \psi_{r(q)} i_{s(d)}) - \frac{1}{I} m_0 \tag{5}$$

The study concerns the stability of the solutions according to the Lyapunov equations (1-5) in the vicinity of their equilibrium states. According to the criterion of Lyapunov direct method, the point of balance is stable if the system trajectory starting close enough to the point of balance and still remains close to that point. For just adopted a mathematical model engine it will continue a three-phase dealt with it in a steady state, was adopted and the load torque m_0 and ω_r the value of the data. According to this system it was adopted $\psi_{r(d)} = const$, and $\psi_{r(q)} = 0$, and after simple transformations we obtain [3]:

$$\psi_{r(d)} = L_m i_{s(d)} \tag{6}$$

$$m_0 = \frac{L_m^2}{L_r} i_{s(q)} i_{s(d)} \tag{7}$$

$$u_{s(q)} = \frac{(L_m^2 + w_\sigma)\omega_r}{L_r} i_{s(d)} + \frac{R_s L_r^2 + R_r L_m^2 + R_r w_\sigma}{L_r^2} i_{s(q)} \tag{8}$$

$$u_{s(d)} = R_s i_{s(d)} - \frac{\omega_r w \sigma}{L_r} i_{s(q)} - \frac{w_\sigma R_r}{L_r^2} \frac{i_{s(q)}^2}{i_{s(d)}} \tag{9}$$

1. $\quad i_{s(d)} = 0 \qquad i_{s(q)} = \dfrac{L_r^2}{R_s L_r^2 + R_r L_m^2 + R_r w_\sigma} u_{s(q)}$ (10,11)

2. $\quad i_{s(d)} = \dfrac{2L_r}{(L_m^2 + w_\sigma)\omega_r} u_{s(q)} \qquad i_{s(q)} = \dfrac{-L_r^2}{R_s L_r^2 + R_r L_m^2 + R_r w_\sigma} u_{s(q)}$ (12,13)

The first solution is rejected, due to the zero stator current $i_{s(d)} = 0$. For further analysis only the second solution is assumed. The designation of the stable operation of the hybrid propulsion system requires determining the values of the individual transmittances that meet the requirements of the criterion adopted. The mathematical model of the object makes it possible to investigate the stability analysis. We can solve a differential equation to yield a solution or using specific criteria for the stability necessary to determine whether there is a solution of the differential equation without solutions. For this purpose, we use the Hurwitz criterion (for linear systems) or Lyapunov criterion (for nonlinear systems [3]).

Own research

The wheeled vehicles with hybrid power units are internal combustion engine and electric motor. Control the operation of both the electric motor and internal combustion engine is carried out automatically. Therefore, the control algorithm should take into account the hybrid automatic adjustment of these two units.

In Fig. 2 is a diagram of a hybrid powertrain. Each of the hybrid drive systems consists of a primary energy source (the work is an internal combustion engine), of a traction motor driving a wheeled vehicle and of a mechanical gearbox. In the case of internal combustion engine and electric motor it is also required to apply an electric generator powered by internal combustion engine. The use of planetary gear allows the synergy of power from both electric motor and internal combustion engine.

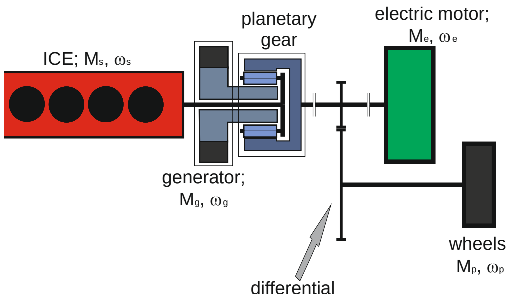

Fig. 2. Diagram of the hybrid powertrain

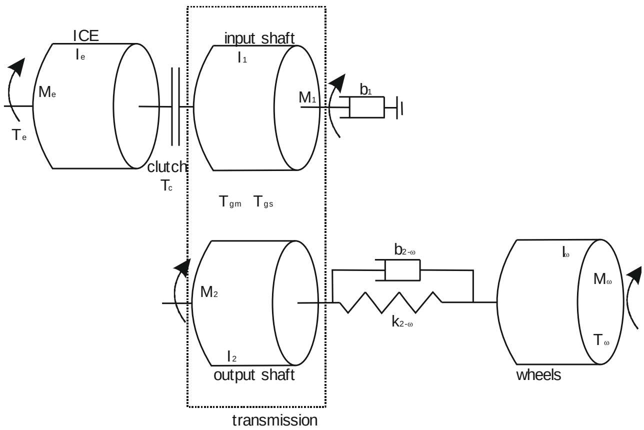

Fig. 3. The structure of the kinematic system block of the drivetrain

The present study focuses on the analysis of the kinematic system stability studies, consisting of an internal combustion engine, mechanical transmission, drive axles and wheels of the vehicle (Fig. 3).

Electric drivetrain

The purpose of the electrical control of the propulsion system is to provide a trajectory mapping engine speed ω , and the flux of the electromagnetic field ψ . Having a mathematical model of the electric motor (1-5), simplification (6-9) and solution (10-13) assumes that $\psi_{r(d)}$, ω trajectories are asked, but I , m_0 have a constant value.

In the adaptive control of follow-up control of flux regulation, we obtain:

$$B_0 \frac{de_\psi}{dt} = B_0 \frac{d\psi_{(d)}}{dt} - v_\psi + e_\psi v_\omega + B_1(\psi^2_{r(d)} + \psi^2_{r(q)}) \tag{14}$$

where: $v_\psi = \psi_{r(d)} i_{s(d)} + \psi_{r(q)} i_{s(q)}$.

To assess the dynamic stability of the system by Lyapunov theory, was used the following Lyapunov function:

$$V = \frac{1}{2}\left[Ie^2_\psi + B_0(\psi^2_{r(d)} + \psi^2_{r(q)})\right] \tag{15}$$

With so adopted system of differential equations and Lyapunov functions can be adopted to determine the stability of the system.

ICE drivetrain

A mathematical model of the kinematic system is composed of a wheeled vehicle with ICE engine (or engines in hybrid vehicle), clutch, drivetrain, and drive axles. The diagram below shows illustrative kinematics of a wheeled vehicle:

$$I_e \dot{\omega}_e = T_e - T_c \tag{16}$$

$$I_1 \dot{\omega}_1 = -b \dot{\omega}_1 + T_c - T_{g(m)} \tag{17}$$

$$I_2 \dot{\omega}_2 = T_{g(s)} - T_{2\omega} \tag{18}$$

$$I_\omega \dot{\omega}_\omega = T_{2\omega} - T_\omega \tag{19}$$

$$\dot{\theta}_{2\omega} = \omega_2 - \omega_\omega \tag{20}$$

where: I_e – interia of the ICE, $\dot{\omega}_e$ – derivative of angular velocity over time, T_e – amount of resistance to motion occurring in the engine, T_c – resistance to motion associated with the coupling, $T_{g(m)}, T_{g(s)}$ – moment of resistance with transmission.

A mathematical model of the kinematic system of a wheeled vehicle for investigating the stability by Lyapunov theorem, required to build Lyapunov functions, which for nonlinear equations may take the form:

$$V = \sum_{j=1}^{n}\sum_{i=1}^{n} A_{ij} x_i x_j + \sum_{k=1}^{m} B_k \int_0^{x_k} f_k(u)du \tag{21}$$

where the coefficients A_{ij} and B_k chosen so as to achieve a positive set V .

For large nonlinearity, certain simplicity in equations (16-20) is possible. To describe the equations of motion are used Lagrange equation:

$$L = \frac{1}{2} I_e \dot{\omega}_e^2 + \frac{1}{2} I_z \dot{\omega}_z^2 = 0 \tag{22}$$

where: I_z – reduced torque on the motor shaft, $\dot{\omega}_z$ – reduced derivative speed of the motor shaft.

Looking for a Lyapunov function according to equation (21), the basis for $\dot{\omega}^2 = x_1$:

$$V = \frac{1}{2} x_1^2 \tag{23}$$

$$\dot{V} = x_1 \dot{x}_1 \tag{24}$$

$$\dot{V} = x_1 (\frac{1}{2} I_e x_1 + \frac{1}{2} I_z x_1) \tag{25}$$

The expression is constant, so the system is stable.

Fig. 4 shows the results; test circuit was built and simulated in the software Matlab/Simulink. Fig. 4 (unstable system) shows the engine control system for which under the influence of external forces the system does not return to its point balance. The internal combustion engine works in this case unstable, the trajectory of the function goes beyond the area of stability (still then grows). In the second case (stable system) are included in the model Lyapunov function. The system is stable, without going outside the approved hyperspherical area ε .

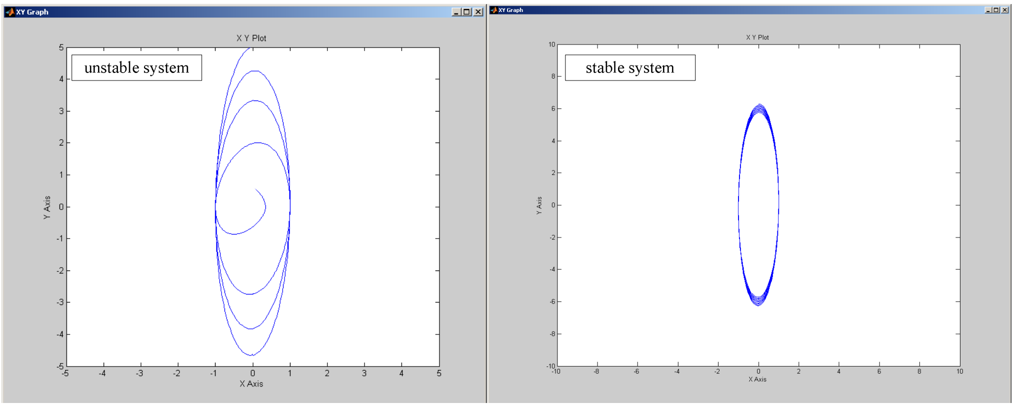

Fig. 4. Simulink results – unstable and stable system

Conclusions

The paper presents a test of stability of a wheeled vehicle powertrain. In the propulsion system for the reproduction of angular velocity "observatory" appropriate speed is used. In any propulsion system may be the occurrence of certain points of instability in the process control. By analyzing the Lyapunov stability method it is possible to give some relationships between the stator current vector in the *d*-*q* system.

In addition, control of a wheeled vehicle takes into account: the type of propulsion, the vehicle kinematics. These factors are to some extent been addressed in the work and the calculation takes into account only some of them, assuming the other parameters as constants. The paper presents the kinematics of wheeled vehicle traffic and made simple mathematical transformations to simplify the mathematical tools. Lyapunov function is determined that gives the differential equation which has a solution - it is stable. The stability of solutions was confirmed by simulating the kinematic model of the hybrid wheeled vehicle in the software Matlab/Simulink. It follows that the system controlling the internal combustion engine and the entire kinematic system of a wheeled vehicle is designed properly.

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